

IMPORTANT JEE-NEET FORMULAS

Vector's Formula's

- **Position Vector Of a Point:** If \vec{a} and \vec{b} are positive vectors of two points A and B, then

$$\vec{AB} = \vec{b} - \vec{a}$$

- Distance Formula:** Distance between the two points $A(\vec{a})$ and $B(\vec{b})$ is

$$AB = |\vec{a} - \vec{b}|.$$

- Section Formula:**

$$\vec{r} = \frac{n\vec{a} + m\vec{b}}{m+n}, \quad \text{Midpoint of } AB = \frac{\vec{a} + \vec{b}}{2}$$

- **Scalar Product of Two vectors:** $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$, where $|\vec{a}|, |\vec{b}|$ are magnitude of \vec{a} and \vec{b} respectively and θ is the angle between \vec{a} and \vec{b}

- $i \cdot i = j \cdot j = k \cdot k = 1$; $i \cdot j = j \cdot k = k \cdot i = 0$, projection of \vec{a} on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$.

- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ & $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.

- The angle ϕ between \vec{a} & \vec{b} is given by $\cos \phi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$, $0 \leq \phi \leq \pi$.

- $\vec{a} \cdot \vec{b} = 0 \leftrightarrow \vec{a}$ Perpendicular to \vec{b} ($\vec{a} \neq 0, \vec{b} \neq 0$).

- **Vector Product of Two vectors:**

- If \vec{a} & \vec{b} are two vectors and θ is the angle between them then $\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \vec{n}$, where \vec{n} is the unit vector perpendicular to both \vec{a} & \vec{b} such that $\vec{a}, \vec{b}, \vec{n}$ forms a right handed screw system.

- Geometrically $|\vec{a} \times \vec{b}| = \text{area of the parallelogram whose two adjacent sides are represented by } \vec{a} \text{ \& } \vec{b}$.

- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$; $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$

- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ & $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

- $\vec{a} \times \vec{b} = \vec{0} \leftrightarrow \vec{a}$ and \vec{b} are parallel (collinear) ($\vec{a} \neq 0, \vec{b} \neq 0$) i.e. $\vec{a} = K \vec{b}$ where K is a scalar.

- Unit vector perpendicular to the plane of \vec{a} & \vec{b} is $\hat{n} = \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$.

vii. If \vec{a}, \vec{b} & \vec{c} are the position vectors of 3 points A, B & C then the vector area of triangle $ABC = \frac{1}{2} [\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}]$. The points A, B & C are collinear if $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

viii. Area of any quadrilateral whose diagonal vectors are \vec{d}_1 & \vec{d}_2 is given by $\frac{1}{2} |\vec{d}_1 \times \vec{d}_2|$.

ix. Lagrange's Identity: $(\vec{a} \times \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix}$

• **Scalar Triple Product:**

i. The scalar triple product of three vectors \vec{a}, \vec{b} & \vec{c} is defined as:

$$\vec{a} \times \vec{b} \cdot \vec{c} = |\vec{a}| |\vec{b}| |\vec{c}| \sin \theta \cos \phi$$

ii. Volume of tetrahedron $V = [\vec{a} \cdot \vec{b} \cdot \vec{c}]$

iii. In a scalar triple product the position of dot and cross can be interchanged i.e.

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c} \quad \text{Or} \quad [\vec{a} \vec{b} \vec{c}] = [\vec{b} \vec{c} \vec{a}] = [\vec{c} \vec{a} \vec{b}]$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = -\vec{a} \cdot (\vec{c} \times \vec{b}) \quad \text{i.e.} \quad [\vec{a} \vec{b} \vec{c}] = -[\vec{a} \vec{c} \vec{b}]$$

iv. If $\vec{a} = a_1i + a_2j + a_3k$; $\vec{b} = b_1i + b_2j + b_3k$ & $\vec{c} = c_1i + c_2j + c_3k$ then

$$[\vec{a} \cdot \vec{b} \cdot \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

v. If $\vec{a}, \vec{b}, \vec{c}$ are coplanar $\leftrightarrow [\vec{a} \vec{b} \vec{c}] = 0$.

vi. Volume of tetrahedron OABC with O as origin & A(\vec{a}), B(\vec{b}) and C(\vec{c}) be the vertices = $|\frac{1}{6} [\vec{a} \vec{b} \vec{c}]|$.

vii. The position vector of the centroid of a tetrahedron if the pv's of its vertices are $\vec{a}, \vec{b}, \vec{c}$ & \vec{d} are given by $\frac{1}{4} [\vec{a} + \vec{b} + \vec{c} + \vec{d}]$.

• **Vector Triple Product:**

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}, \quad (\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$$

$$\text{In general: } (\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$$